Effect of Angle of Incidence on the Response of Cylindrical Electrostatic Probes at Supersonic Speeds

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The response of cylindrical probes at various angles of incidence has been studied in low-density hypersonic argon plasma. All mean free paths were larger than probe radii (r_p) except the ion-ion mean free path which was smaller than r_p in most of the tests. In nearly all experiments, an ion current peak has been observed near the zero angle of incidence. The magnitude of this effect is shown to depend strongly on probe radius-to-Debye length ratio (r_p/λ_D) and is significant for $0.5 < r_p/\lambda_D < 2$. Insensitivity of this effect to probe length appears to be closely related to the presence of ion collisions. A physical model is proposed which explains the effect in terms of ion-neutral collisions in the presheath of the probes. A simple theory is presented in the paper which attempts to account for the measured data on the ion current peak. The readings obtained with aligned and transverse probes are interpreted using collisionless theories (Laframboise, Smetana, modified Langmuir) and the current vs angle-of-attack variation is used to infer "collision-free" value of the ion current to aligned probes. The experimental results appear to agree with Laframboise's theory in the orbital-motion-limited region. Disagreements with theory reported by several investigators can be explained either in terms of ion-neutral collisions associated with aligned probes (in "nearly collisionless" plasmas) or in terms of an end effect (in highly rarefied plasmas).

1. Introduction

THE behavior of thin-wire electrostatic probes in stationary and collision-free plasmas is fairly well understood. However with the advent of satellites, magneto-plasma-dynamic accelerators, and high-temperature hypersonic wind tunnels, there has been a marked increase in plasma research where the velocity relative to the probe exceeds the thermal velocity of ions. For such flow conditions, there are many aspects of electrostatic probe response that remain unexplained. One of them is the effect of the angle of incidence of the probe on the ion current collection. Sonin[1] observed an unexpected sharp increase in the ion current when the probe was brought into alignment with the flow velocity. Understanding this phenomenon, which is not predicted theoretically, may be essential for proper interpretation of probe readings under conditions of high-speed flows and intermediate values of probe radius-to-Debye length ratio (r_p/λ_D). In particular, an indiscriminate use of aligned probes with an a priori assumption that the corresponding measurements can be interpreted in terms of the theory available for stationary plasmas, may result in large errors. The main purpose of this study is to investigate the conditions under which an ion current peak occurs near zero angle of attack of a cylindrical probe immersed in a high-speed flow of an ionized gas. In addition, an investigation is made of the probe response and its correlation with theoretical analyses in the range of intermediate ratios of r_p/λ_D ($0.1 < r_p/\lambda_D < 10$), which is the most complex and least understood regime of the probe operation.

Following the original work of Langmuir and Mott-Smith[2] a large number of attempts have been made at developing the theory of current collection by electrostatic probes in collisionless plasmas.[3-6] Currently, the most widely accepted theory is that of Laframboise[6] who obtained a numerical functional analysis based on the microscopic theory of Bernstein and Rabinowitz.[4] The results obtained by Laframboise and most of his predecessors are limited to the case of stationary plasmas. In the general case of a flowing plasma, the potential distribution around the probe loses its symmetry and a rigorous solution of the problem becomes a formidable task. However, if an infinitely long cylindrical probe is directed along the stream, the distortion of the potential field can be avoided and the response of the probe can be, at least in principle, interpreted within the framework of static theory. This approach has been extensively used during the past several years, since an interest in probe measurements in flowing plasmas started to grow rapidly.

Theories of current collection by cylindrical probes in flowing, collisionless plasmas have been published by Smetana[7] and Kanal.[8] These authors assumed that the electric field asymmetry produced by the motion of plasma had only a slight effect on the current collection. The theory of the Langmuir probe behavior in the transition regime is much less developed and most of the analyses deal with the spherical probes. Recently, Talbot and Chou[9] presented an approximate analysis of the effect of collisions for both cylindrical and spherical probes that agrees reasonably well with experimental data.

Several investigators performed experiments with thin-wire probes in flowing low-density plasmas.[1,10-17] Sonin[1] carried out the first systematic study of cylindrical probes in supersonic plasma flows. His results seemed to confirm Laframboise's theory in the range of $r_p/\lambda_D > 2.8$ but departed significantly from the theory in the orbital-motion-limited (OML) region.† Lederman, Bloom, and Widhopf[14] reported strong departures from Laframboise's theory, a significant length-to-diameter ratio effect below $l_p/d_p = 225$, and a strong but inconclusive effect of the angle of incidence. In contrast to findings of the previously published data,[1,11,14] the experimental results of Dunn and Lordi[15,16] confirmed Laframboise's theory in at least a small portion of the OML region (for $r_p/\lambda_D > 1$).

Despite a significant progress in Langmuir probe diagnostics of flowing plasmas, there is a lack of agreement among the results

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† Recently, Hester and Sonin[18] reexamined Sonin's results and concluded that a) ion current increase observed with aligned probes was caused by small ion-ion mean free path and b) the results could be brought into an agreement with the theory of Allen-Boyd-Reynolds instead of that of Laframboise.

obtained by several investigators who studied wire probes in supersonic plasma streams.[1,11,13-15,18] In particular there remains a substantial uncertainty concerning: a) the behavior of the probes in the OML region, and b) the influence of the angle of attack. The second question may be, under some particular conditions (ionospheric conditions, highly rarefied plasma), related to an end effect (finite-length effect).[18,19] The ion current peak at zero angle of attack, observed by Sonin and in these experiments has been found insensitive to probe length and, therefore, its origin must be sought elsewhere.

In experiments reported here, cylindrical probes are investigated in a hypersonic low-density argon plasma stream. The effect of the angle of incidence on the current collection is found to be very strong in the range of intermediate values of r_p/λ_d. The effect which amounts to occurrence of a sharp ion current peak near zero angle of attack can be explained in terms of ion-neutral collisions in the region of small electric field outside the probe sheath. Experimental data are compared with theoretical predictions based on a simple approach presented in this paper.

Evaluation of ion number density is based on collisionless theories (Laframboise, Smetana, modified Langmuir) and discrepancies between some previously reported experimental data and Laframboise's theory are explained in terms of ion-neutral collisions.

2. Experimental Apparatus

Experiments were performed in the low-density plasma jet facility of the North Carolina State University. Details of this facility have been reported in Ref. 20. The facility used an arc-heated hypersonic freejet expanded from a supersonic nozzle. Argon of commercial grade (99.998% purity) was used as the test gas. The experimental set-up for probe investigation is shown schematically in Fig. 1. Electrostatic probes were mounted on an assembly which included a 3-dimensional traversing mechanism. The latter allowed for a parallel and a transverse motion with respect to the flow axis, and a rotation about a horizontal axis (through about 150°). Probes were mounted in arrays of up to three at a time and were adjusted so that the centers of the exposed wires fell on the axis of rotation of the rotating gear (within ± 1 mm). The distance between any two probes was usually kept at least 20 mm to reduce any possible interactions.

Probes used in this investigation employed tungsten wires as collectors of charged particles and Pyrex glass as tubular shields. The diameter of the wire ranged from 0.025 mm to 0.254 mm and the length-to-diameter ratio was varied between 35 and 645. Details of probe construction and operation have been discussed in Ref. 21. The basic circuit for sweeping the probe voltage and recording the current-voltage characteristics is shown schematically in Fig. 1. Usually, the range of characteristics covered by the voltage sweep included the ion current region and a portion of the electron retarding-field region.

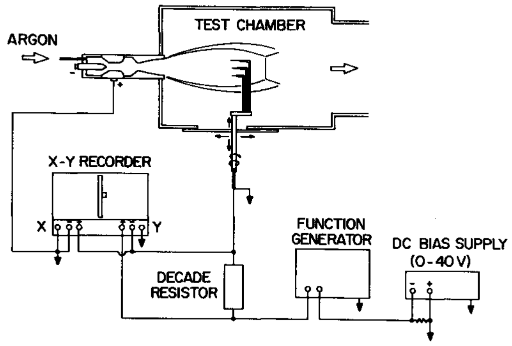

Fig. 1 Experimental set-up for Langmuir probe measurements.

At the beginning of each run, probes to be used were cleaned by applying a large negative voltage (200–300 v) which resulted in ion bombardment. After cleaning, the probe characteristics were usually very reproducible.

The test conditions in this investigation were as follows: $0.1 < r_p/\lambda_D < 15$, $0.1\ \text{ev} < T_e < 0.5\ \text{ev}$, $\sim 7°\text{K} < T_i < 54°\text{K}$, $7 \times 10^8 < n_i < 7 \times 10^{11}\ \text{cm}^{-3}$, $1.6 < S < 3$, $15 < M_{\text{isen}} < 24$, $0.05 < \lambda_{ii}/r_p < 10$, $0.7 < \lambda_{in}/r_p < 75$, $30 < \lambda_{nn}/r_p < 900$, and $80 < \lambda_{ee}/r_p < 2.5 \times 10^4$; where T_e and T_i are the electron and ion temperatures, $S = u/(kT_e/m_i)^{1/2}$ is the ion acoustic Mach number (u = flow speed, m_i = ion mass), M_{isen} is the isentropic Mach number, and λ is the mean free path (i = ion, n = neutral, e = electron). Stagnation pressure and temperature ranged from 55 to 167 torr and from 850 to 5050°K, respectively.

3. Procedures

Experimental Procedures

A usual test procedure was to plot a current-voltage curve for each of the preselected angles of incidence of a probe. At very low plasma densities, an additional procedure was employed which consisted of applying a constant voltage to the probe, and varying its angle of attack in prescribed steps, with the ion current being recorded simultaneously. In order to establish an effective zero angle of incidence ($\theta = 0°$), the readings were taken over a range of θ including both positive and negative values of θ.

An important aspect of measurements in a freejet is a selection of a proper location on the jet axis. A location upstream of the ion shock wave (which could be identified by electrical diagnostics of the freejet[21]) was found to be the most appropriate and was characterized by small longitudinal and radial gradients in ion density and freedom of diffusional background effects.

During each experiment with Langmuir probes, several electrical and gasdynamic measurements were performed to provide or supplement information on the conditions of the experiment.[20]

Probe Response Interpretation

The electron temperature T_e was determined for each run from the electron retarding-field range of the current-voltage characteristic (semilog method). Usually the logarithm of the electron current plotted vs the probe potential yielded a straight line extending over at least one decade. At very low plasma densities, the straight-line portion tended to shrink which might increase the usual experimental uncertainty in defining T_e.

The ion-electron number density ($n_i = n_e$) was calculated from the ion current measured at the dimensionless probe potential $\eta_p = e(V_p - V_\infty)/kT_e = -15$ (V_p = probe potential, V_∞ = plasma potential). Two essentially independent determinations of n_i were performed for each test. For an aligned-probe orientation, which under some conditions may simulate static conditions, the theory of Laframboise was applied and a value of ion current was used that was inferred (as it will be explained in Sec. 4) from the experimental plot of ion current vs angle of attack, $I_p = f(\theta)$. For negative values of η_p, the ion current density can be expressed in the form

$$j_i = (en_i/4)(8kT_e/\pi m_i)^{1/2} I_i(\eta_p, r_p/\lambda_D, T_i/T_e) \tag{1}$$

where $\lambda_D = (\varepsilon_o kT_e/e^2 n_e)^{1/2}$ is the electron Debye length, and I_i is dimensionless ion current which gives the increase in the current collected by the probe over the random kinetic flux at $\eta_p = 0$ (i.e., when the probe is at the plasma potential). Laframboise calculated the magnitudes of I_i for a wide range of plasma and probe parameters.

For the transverse-probe orientation, an approach was required which could account for the high-speed motion of the plasma. The theory of Smetana and a simple modification of Langmuir's formula were applied for this case. Smetana obtained a set of numerical solutions which can readily be used when 1) the probe axis is perpendicular to the plasma stream, 2) Debye length is larger than the probe diameter ($\lambda_D > d_p$), 3) ion speed ratio $u/(2kT_i/m_i)^{1/2}$ is large, and 4) the negative probe potential is not very large.

Langmuir's modified formula can be derived from the equations of orbital motion of ions under the assumptions that a) the sheath is thick ($r_s \gg r_p$, see Fig. 2), b) the probe is negatively

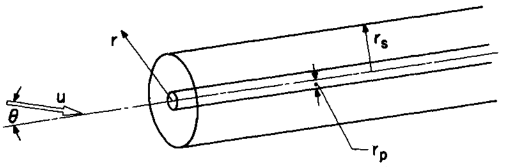

Fig. 2 Idealized sheath about a cylindrical probe.

biased, and c) the flow speed is very large compared with the thermal ion speed, $u/(2kT_i/m_i)^{1/2} \gg 1$, and consequently, the ions can be treated as monoenergetic. For such conditions, the "absorption" impact parameter is $p_a = r_p(1-2eV_p/m_i u^2 \sin^2\theta)^{1/2}$ and the current to the probe of the length l_p becomes

$$I_p = en_i d_p l_p u \sin\theta[1-2eV_p/m_i(u\sin\theta)^2] = en_i u d_p l_p(\sin^2\theta + 2\eta_p/S^2)^{1/2} \quad (2)$$

This simple modification of an expression given by Langmuir[2,19] allows for various orientation of the probe relative to the flow direction. For the transverse and parallel probe orientation one obtains

$$I_{p_\perp} = en_i u d_p l_p(1+2\eta_p/S^2)^{1/2} \quad (3)$$

$$I_{p_\parallel} = en_i u d_p l_{p_\infty}(2\eta_p)^{1/2}/S \quad (4)$$

where an additional subscript "∞" is used at l_{p_∞} to emphasize that the expression (4) is appropriate for an "infinitely" long probe only. Throughout this investigation, Eq. (3) was used for the evaluation of n_i from the transverse-probe response. In all cases the results agreed closely with those derived from Smetana's theory as could be expected since both methods are based on similar concepts of orbital motions and the assumption of monoenergetic ions incorporated in Eq. (3) is not restrictive at very low ion temperatures.

The $I_p = f(\theta)$ dependence represented by Eq. (2) indicates a monotonic increase of the ion current with the angle of attack. Because of the assumption c, Eq. (2) can be approximately correct only for

$$u \sin\theta \gg (2kT_i/m_i)^{1/2}$$

i.e., only in the range of relatively large angles θ. At a given negative probe potential, the absorption parameter p_a is approximately inversely proportion to the flow velocity component $u\sin\theta$, while the flux density of ions, $enu\sin\theta$, is directly proportional to $u\sin\theta$. Consequently, at very high speeds, these two effects tend to cancel each other [see Eq. (2)] and the ion current collected by a probe is 1) only slightly different from the current collected in the same plasma at zero velocity and 2) weakly dependent on θ with the exclusion of θ near zero angle of attack [i.e., for $u\sin\theta \gg (2kT_i/m_i)^{1/2}$]. This is expected to be true only when $r_s \gg r_p$ and the negative potential of the probe is fairly high. When the sheath is thin and the flow velocity is very high, then the current collected is approximately equal to the flux of ions swept by the probe [Eq. (2) with $u \to \infty$]

$$I_p \simeq en_i d_p l_p u \sin\theta \qquad (r_s \approx r_p)$$

and the effects of velocity and probe orientation may be quite significant.

Evaluation of Gasdynamic Properties of Plasma

Flow velocity u, isentropic Mach number M_{isen}, ion acoustic Mach number S, and ion temperature T_i were estimated on the basis of gasdynamic measurements, electron temperature, and an assumption that the plasma entering the supersonic nozzle was in thermal equilibrium. This assumption received strong support from experimental and analytical examination of the termal equilibrium in the plenum chamber for the conditions typical in Langmuir's probe tests.[20]

In order to arrive at possibly realistic estimates of ion temperature in the expanded jet, which were necessary for evaluation of the mean free paths λ_{ii} and λ_{in}, possible departures from isentropic temperatures were considered. As collisions become less numerous far downstream of the nozzle, an increasing anisotropy in random velocity distribution is likely to occur and this effect may be described in terms of two translational temperatures, one along streamlines, $T_\parallel$, and the other transverse to streamlines, $T_\perp$. The parallel temperature, $T_\parallel$, freezes gradually while the perpendicular one, $T_\perp$, continues to decrease isentropically for much larger distances. For the purpose of this investigation, the theory of Hamel and Willis[22] was applied. Using their solutions of a hypersonic approximation to the moment equations and assuming Maxwell molecules (inverse fifth power), the parallel temperature was calculated as the one which was probably the most important in these experiments.

The ion-ion mean free path, λ_{ii}, was calculated as the distance traveled by a test particle while gradually accumulating a 90° deflection.[23] The theory on which this calculation is based breaks down at higher densities and low temperature. Such conditions occurred during several runs and the values of λ_{ii} inferred by extrapolation were then assumed. Evaluation of the ion-neutral mean free path, λ_{in}, was based on the collision cross section for the momentum transfer because this effect could possibly influence the ion current collection, as shown in Sec. 4. At the densities and energies encountered in this study, only a very small fraction of ion-neutral collisions involved charge transfer effects. The momentum transfer cross section can be related to the mobility through the ion-neutral mutual diffusion coefficient. For a temperature range $T_i > 50$°K, the values of argon ion mobility in the parent gas, determined by Chanin and Biondi,[24] were used. For ion temperatures $T_i < 50$°K, a constant value of the reduced mobility was assumed, equal to 2.1 cm^2/v-sec (Refs. 21 and 25).

4. Results and Discussion

Initial experiments showed very clearly that the variation of the ion current with the orientation of the probe is strongly influenced by the ratio of the probe radius to Debye length r_p/λ_D (Fig. 3).

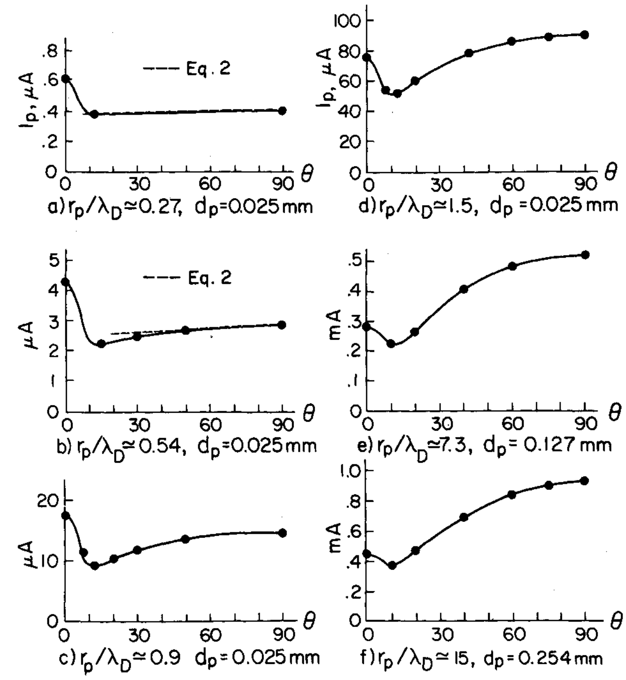

Fig. 3 Variation of ion current with orientation at various r_p/λ_D, $\eta_p \approx -15$, $19 < M_{\text{isen}} < 22$, $2 < u/(kT_e/m_i)^{1/2} < 2.5$.

As the probe was turned from a perpendicular position ($\theta = 90°$), the ion current initially decreased, thus following a trend expected theoretically, but then, as the probe approached a position aligned with the flow, the ion current increased and reached a relative maximum at $\theta = 0°$. The "height" of the ion current peak was the largest in the range $0.5 < r_p/\lambda_D < 1$, and it decreased with an increase in r_p/λ_D vanishing completely at sufficiently large r_p/λ_D. The peak in current decreased also below $r_p/\lambda_D \approx 0.5$; however, the corresponding experimental data was somewhat less conclusive and could not be extended to less than $r_p/\lambda_D \approx 0.15$. The experimental results shown in Fig. 3a–b were obtained when all mean free paths were larger than r_p and $r_s/r_p \gg 1$. For such conditions experimental data on angular dependence $I_p = f(\theta)$ can be compared with the theoretical values predicted by Eq. (2), indicating fair agreement within the range of applicability of Eq. (2), i.e., for $u \sin\theta \gg (2kT_i/m_i)^{1/2}$. When r_p/λ_D was larger than about 0.7, λ_{ii} was always smaller than the probe radius; however, all the remaining mean free paths were larger than r_p. A typical contour of the ion current peak is shown in Fig. 4. The angle of incidence corresponding to a minimum value of the ion current varied between 10° and 16°.

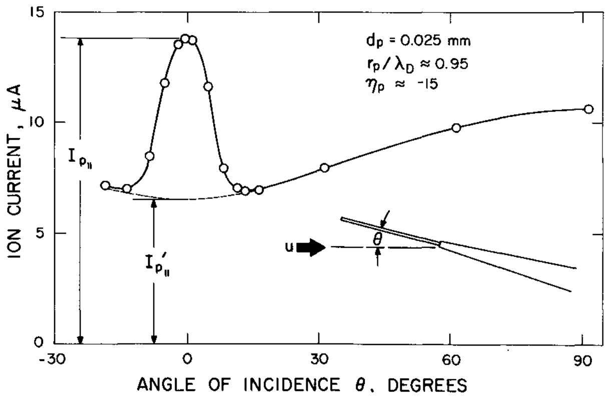

Fig. 4 Variation of ion current with probe orientation.

In order to determine whether the ion current peak was sensitive to probe length, tests were made using probes of the same diameter and of different lengths. The results are presented in Fig. 5 which shows ion current density measured at a fixed potential ($\eta_p \simeq -15$) and normalized by the value corresponding to the probe with the same orientation and the maximum value(s) of l_p/d_p. It can be observed that even a probe with length-to-diameter ratio of 35 did not display any significant finite length effect. Insensitivity to probe length was found to be independent of r_p/λ_D.

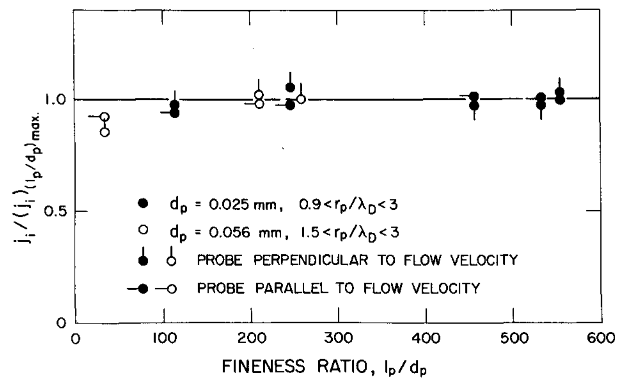

Fig. 5 Variation of ion current density with fineness ratio.

Once the end effect was excluded as a possible cause of the ion current peak at $\theta = 0°$, the main attention was centered on ion collisions and their influence in a regime generally considered collisionless. If we assume that the entire ion current peak is due to some collisional effects, then the value of $I'_{p_{\parallel}}$ obtained by an "extrapolation" of the right portion of the curve $I_p = f(\theta)$ (i.e., the part of the curve presumably unaffected by collisions) should correspond to the ion current, that would be obtained in a flow free of any collisional effects. Such extrapolation is indicated by a broken line in Fig. 4. Using the value of $I'_{p_{\parallel}}$ in a well established theory for wire probes in a stationary plasma,[6] one should arrive at an approximately correct value of the ion number density. The value of $n'_{i_{\parallel}}$, so obtained can be compared with the value of $n_{i_{\perp}}$ derived from the measured value of I_p at $\theta = 90°$, $I_{p_{\perp}}$, by applying Eq. (3) or calculations of Smetana. This procedure has been applied to a number of probe readings and the results are shown in Fig. 6. In several cases, the extrapolation

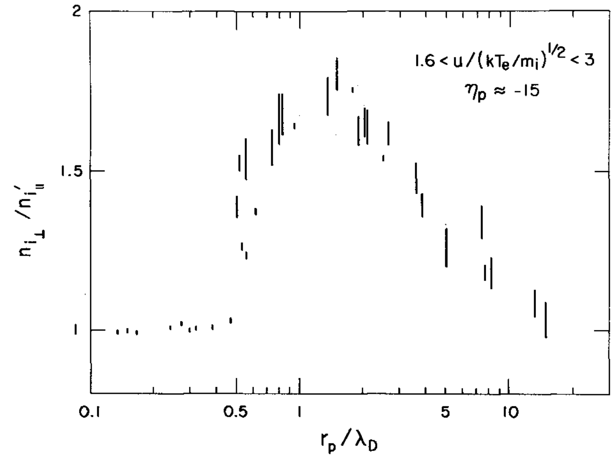

Fig. 6 Ratio of ion density inferred from perpendicular probe orientation to that from parallel probe orientation as a function of r_p/λ_D.

aimed at defining $I'_{p_{\parallel}}$ has been somewhat arbitrary leading to a range of values of $I'_{p_{\parallel}}$ rather than to a one specific value. In the range of $r_p/\lambda_D < 0.5$, the agreement between $n'_{i_{\parallel}}$ and $n_{i_{\perp}}$ is quite good. (Note that at $d_p/\lambda_D < 1$, and $T_i/T_e \ll 1$, all three methods use similar concepts of orbital motions.) In this range the electric field penetrates far from the probe surface and, over the most of the field, the potential falls off less rapidly than r^{-2}. Consequently, with no collisions present, ions can move in trajectories not impeded by intermediate barriers of effective potential i.e., potential energy including repulsive contribution of angular momentum, and the response of the probe should be described correctly by the concepts of OML current collection provided that the influence of potential wells (with trapped orbits), unaccounted for by theory, can be neglected. The population of trapped orbits tends to increase when the potential falls slower than r^{-2}. However, if the plasma is streaming parallel to the probe axis, the trapped ions will be washed away and it may be argued that the response of a long cylindrical probe aligned with the flowing plasma can be described more correctly by the available static theory than the response of the same probe in a stationary plasma. An additional argument in support of the validity of simple orbital theory is connected with the very low ion temperature ($T_i < 15°K$) in the experiments at $r_p/\lambda_D < 0.5$. Because of this low temperature, the plasma stream was nearly monoenergetic and the current collection could possibly remain orbital-motion-limited for larger values of r_p/λ_D than it would be possible at large T_i. On the basis of these considerations, it has been concluded that a) the ion current peak at $\theta = 0°$ was indeed produced by collisional effects, and b) the theory of Laframboise predicts correctly current collection in the OML region.

As r_p/λ_D increases above 0.5, the value of $n_{i_{\perp}}$ becomes larger than $n'_{i_{\parallel}}$, which means that, at $r_p/\lambda_D > 0.5$, Eq. (3) (and Smetana's theory) overestimates the ion density. This, of course, can be expected from an approach (modified Langmuir, Smetana) which does not account for existence of intermediate barriers in effective potential.

An additional indication that the probe response followed Laframboise's predictions was provided by the current measurements during runs in which two or three probes of different diameters, placed simultaneously in the plasma stream, were subject to operation ranging from orbital-motion-limited to "space-charge-limited." This can be seen from Fig. 7 where the dimensionless ion current I_i, defined by Eq. (1), is plotted vs

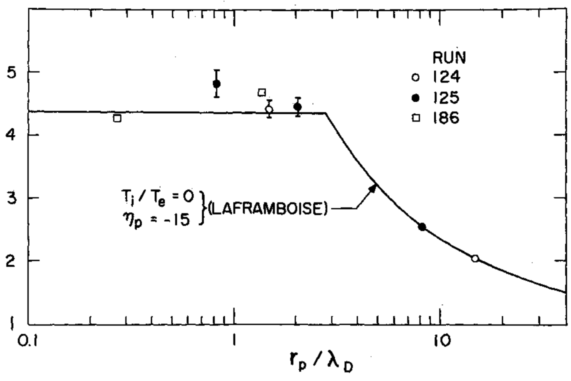

Fig. 7 Dimensionless ion current vs r_p/λ_D.

r_p/λ_D. Using the "bootstrap" technique similar to the one used by Sonin,[1] the experimental points in the range of $r_p/\lambda_D > 2.8$ were adjusted so as to match the theoretical curve for $T_i/T_e = 0$. For the run 186 in which two probes operated in the OML region, the value of I_i of the thinner probe was adjusted so as to match the values of $n'_{i_\parallel}$ and $n_{i_\perp}$ (note that at $r_p/\lambda_D < 0.5$, the values of $n_{i_\perp}$ are believed to be approximately correct). It can be recognized that the dimensionless ion current tends to level off in the OML region.

A fairly large scatter of the data shown in Fig. 6 can be attributed to the fact that experiments were made at different stations along the jet and the influence of the ion Mach number and ion temperature was not isolated in the presentation of the data. It is, of course, recognized that the ion acoustic Mach number, S, must have an influence on the structure of the ion current peak. However, in these experiments S varied over a relatively narrow range ($1.6 < S < 3$), and its influence could be easily obscured by the scatter of the data. In a discussion which follows, a physical model is presented which accounts for the ion current peak near zero angle of attack.

Physical Model

When a long cylindrical probe, negatively biased w.r.t. the plasma potential, is aligned with a flow of rapidly moving cold ions, an average ion entering the presheath region (a large quasi-neutral transition region between space charge sheath and "free-stream" undisturbed plasma) may travel a long distance within this region before it is collected by the probe or is scattered out of the field of attraction. If there are no collisions within this region, such an ion, according to its individual energy and angular momentum, may either enter the space charge sheath and become collected by the probe or it may leave completely the field of the probe attraction. If, however, an average ion is subject to collisions in the presheath region, its chances of escaping are, as it will be shown, substantially reduced.

Consider first ion-neutral encounters. When an ion collides with a neutral particle, it will likely lose a fraction of its kinetic energy (in the presheath, the oncoming ions move, on average, faster than neutrals) and consequently will be captured by the probe. Also, because of a relatively short-range character of an ion-neutral encounter, this process may, under certain conditions, result in a break-down of the shielding (in the presheath, ions are approximately, although not completely, shielded by the electrons). In particular, if the ion-neutral mean free path, λ_{in}, is much shorter than λ_D (in these experiments, $\lambda_{in} < \lambda_D$ in practically all runs at very low ion density (10^8–10^9 cm^{-3}) and small r_p/λ_D), then a collision may result in a complete break-down of the shielding as the electrons cannot immediately shield the ions on a scale smaller than λ_D. An immediate consequence of the shielding break-down is a strong polarization and a rapid acceleration of the ion toward the probe. The mechanism of ion-neutral encounters in the presheath region (kinetic energy loss and/or shielding breakdown) may explain the rise in the ion current of an aligned probe. The process would be equivalent to an effective expansion of the space charge sheath. However, this process may account for an increase in the ion current only as long as an average ion undergoes no collision in the sheath itself. When an ion makes collisions in the sheath, it will be subject to elastic scattering which leads to a decrease in the ion current density. As a result, the aforementioned ion current peak near $\theta = 0°$ may occur over a relatively narrow range of r_p/λ_D. As r_p/λ_D decreases below the value corresponding to the maximum amplitude of the ion current, the ion current peak is first reduced, then cancelled out completely, and finally, the ion current decreases to a collision-dominated value.

Next, consider ion-ion encounters. Ion-ion collisions in the presheath region are not likely to produce any significant effect on the ion current to the probe because such interactions are predominantly weak and essentially do not disturb the shielding. Ion-ion collisions in the sheath are associated with a continuous exchange of the angular momentum and energy between the particles involved. Although the angular momentum of any individual ion is not conserved, nevertheless, it seems possible that there is no net effect of ion-ion collisions on ion collection by the probe, since there is no momentum transfer in the ion gas as well as between the ion gas and other species (provided the flow is in a transition regime w.r.t. the ions only).

As the probe is turned from the zero angle of attack, the distance traveled by an average ion through the presheath zone is reduced drastically and so the chances for ion-neutral encounters within this zone. Even if collisions take place, the angular momentum of an average ion may prevent its capture by the probe. Consequently, a probe which, when aligned with the flow velocity, is, de facto, in the transition regime, after being turned by 90° may behave as in a collisionless regime provided $\lambda_{in} \gg r_p$.

To analytically verify the proposed physical model we first examine the maximum amplitude of the ion current peak when plotted vs r_p/λ_D. This maximum is expected to occur when λ_{in} is somewhat larger than sheath radius r_s, e.g., $r_s < \lambda_{in} < 2r_s$ (note that ion trajectories within the sheath are mostly very steep). If an average ion collides with a neutral particle in the presheath and is subsequently collected by the probe, without making any collisions in the sheath, the situation corresponds essentially to a free fall from the location of the last collision in the presheath. Because at high-hyperthermal velocities, ion trajectories in the presheath are predominantly shallow, the location of the last collision is, on average, close to the sheath edge and, therefore, the current to the probe will be approximately

$$I_{p_\parallel} = en_i u_{s_\perp} 2\pi r_s l_p \tag{5}$$

where $u_{s_\perp}$ is the velocity component normal to the sheath boundary. At $T_i/T_e \ll 1$, the velocity $u_{s_\perp}$ is approximately equal to the ion acoustic speed $(kT_e/m_i)^{1/2}$ and is independent of the tangential velocity.[26] Hence, the amplitude of the ion current peak normalized by a collisionless value (taken from Laframboise) becomes

$$\frac{I_{p_\parallel}}{I'_{p_\parallel}} = \frac{en_i(kT_e/m_i)^{1/2} 2\pi r_s l_p}{(en_i/4)(8kT_e/\pi m_i)^{1/2} 2\pi r_p l_p I_i(\eta_p, r_p/\lambda_D, T_i/T_e)} = \frac{(2\pi)^{1/2}}{I_i} \frac{r_s}{r_p} \tag{6}$$

The ratio $I_{p_\parallel}/I'_{p_\parallel}$ may be considered as a measure of the ion current enhancement due to the collisional effects. The sheath thickness may be estimated by using an equation derived by Bettinger and Walker[27]

$$[(r_s/\lambda_D)^2 + 2(r_p/\lambda_D)(r_s/\lambda_D)] \ln(1 + r_s/r_p) = \pi\eta_p(1 + 2\eta_p/3)^{1/2} \tag{7}$$

and by applying a "scaling factor" (0.5) recommended in Ref. 19. The preceding equation was derived for a static probe and, therefore, it may be appropriate for an aligned orientation. Figure 8 shows the variation of $I_{p_\parallel}/I'_{p_\parallel}$ [Eq. (6)], r_s/r_p, $2r_s/r_p$, and λ_{in}/r_p vs r_p/λ_D. The maximum value of $I_{p_\parallel}/I'_{p_\parallel}$ is expected between the points of intersection of λ_{in}/r_p curve with r_s/r_p and $2r_s/r_p$, i.e., in the range $0.35 < r_p/\lambda_D < 0.55$ (average $r_p/\lambda_D \approx 0.45$) for 0.025-mm-diam probes and in the range $0.6 < r_p/\lambda_D < 0.8$ (average $r_p/\lambda_D \approx 0.7$) for 0.056-mm probes. Corresponding magnitudes of

$(I_{p_\parallel}/I'_{p_\parallel})_{max}$ (for average r_p/λ_D) are 4.2 and 2.9, respectively. It may be noted that if the values of $I_{p_\parallel}$ and λ_D are known Eq. (5) may serve to obtain an estimate of the sheath radius r_s.

When ions make collisions in the sheath ($\lambda_{in} < r_s$), they will be elastically scattered and their average transit time across the sheath will be increased. Of the unscattered ions reaching a distance r from the probe, the number scattered out in the volume dr, per second and per unit cross-sectional area, is (using a simplified one-dimensional approach) $dn = (r_s/\lambda_{in})(1/t_s)n\,dr$, where t_s = transit time of unscattered ion in the sheath and n = number of ions per unit volume. Note that because the path of an unscattered ion in the sheath is very steep during the most part of the transit time, it may be approximated by a radial trajectory. Since the ion current density is $j = nev$, $(v = dr/dt)$, thus $dn = (j/e)(r_s/\lambda_{in})dt/t_s$, and the corresponding depletion of the current of unscattered ions is

$$dj = -e\,dn = -j(r_s/\lambda_{in})dt/t_s$$

Integrating one obtains for the current of unscattered ions

$$j = j_s e^{-(r_s/\lambda_{in})(t/t_s)} = j_s e^{-\xi} \tag{8}$$

where j_s is the current at the sheath edge and $(r_s/\lambda_{in})(t/t_s) = \xi$. Next consider scattered ions reaching the volume dr. With only one or two collisions in the sheath, the low-temperature ions are scattered mostly in the forward direction. The average r-component of the velocity of the scattered ions, $v_\rightarrow$, is equal $v/2$. If the number of forward scattered ions arriving at r is $n_\rightarrow$, the number scattered out per unit time and per unit area is $dn_\rightarrow = (r_s/\lambda_{in})(1/t_s)n_\rightarrow dr$, and the corresponding current depletion is $dj_\rightarrow = -2j_\rightarrow d\xi$. The net change of the forward scattered ion current becomes

$$dj_\rightarrow = j\,d\xi - 2j_\rightarrow d\xi \tag{9}$$

(Note that depletion of unscattered ions means an increase of scattered ones, hence, plus sign appears at j). The current reaching the probe is

$$j_p = j + j_\rightarrow \tag{10}$$

Using Eqs. (10) and (8) in Eq. (9), the latter can be integrated subject to the boundary condition that the current of scattered ions is zero at the sheath edge. The result expressed as the ratio of the current density reaching the probe to that reaching the sheath is

$$j_p/j_s = (3 - 2e^{-\xi_p})/(1 + 2\xi_p) \tag{11}$$

We note that $\xi_p = r_s/\lambda_{in}$ and, therefore, ξ_p represents the number of collisions in the sheath.

If ions make several collisions in the sheath, the scattering tends to become spherically symmetrical and the ions scattered in the backward direction have to be taken into account. For a spherically symmetrical scattering, the average velocity for either the forward or backward scattered ions is $v_{r_{ave}} = \int v_r\,dn/\int dn = v/2$. The losses for: a) the unscattered ion current j passing the volume element dr, b) forward scattered ion current $j_\rightarrow$, and c) backward scattered ion current $j_\leftarrow$, become correspondingly a) $-j\,d\xi$, b) $-2j_\rightarrow d\xi$, and c) $-2j_\leftarrow d\xi$. The net change in the forward scattered ion current is obtained as

$$dj_\rightarrow = \tfrac{1}{2}(j\,d\xi - 2j_\rightarrow d\xi + 2j_\leftarrow d\xi) \tag{12}$$

Plus sign at j and $j_\leftarrow$ indicates that losses in these currents represent gains for $j_\rightarrow$. The coefficient $\frac{1}{2}$ accounts for the fact that in the case of spherically symmetrical scattering the number of ions scattered at any given moment is equally divided between two classes of forward and backward scattered ions. Now

$$j_p = j + j_\rightarrow - j_\leftarrow \tag{13}$$

where j_p is considered constant. Equations (8, 12, and 13) can be solved using the boundary conditions that 1) the current of forward scattered ions is zero at the sheath edge and 2) the current of backward scattered ions is zero at the probe surface. The solution can be written in the form

$$j_p/j_s = (3 - e^{-\xi_p})/2(1 + \xi_p) \tag{14}$$

When the number of collisions in the sheath exceeds approximately 6, the motion of ions becomes governed by mobility and diffusion. Such a case is beyond the scope of this investigation. In order to compare simple theoretical predictions given by Eqs. (11) and (14) with experiment, we note that an estimate of the current reaching the sheath can be obtained from the value of $(I_{p_\parallel}/I'_{p_\parallel})_{max}$ obtained from Eq. (6). Using the values of r_s and λ_{in} plotted in Fig. 8, approximate values of r_p/λ_D can be found,

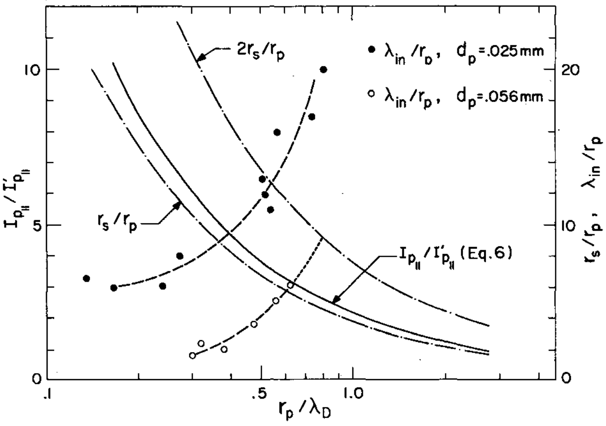

Fig. 8 Ion current peak, sheath radius, and ion-neutral mean free path vs r_p/λ_D.

which correspond to one, two, three,... collisions in the sheath. Equations (11) and (14) are then used to plot theoretical predictions of $I_{p_\parallel}/I'_{p_\parallel}$. Figure 9 shows these predictions along with experimental results obtained in this study. Large quantitative departures between experiment and theory are apparent and they are probably due to large uncertainties in estimates of both r_s and λ_{in} (about $\pm 50\%$ for r_s and a factor of two or three for λ_{in}) and to approximate nature of the analysis. The general trends in the variation of the ion current peak, including the location of $(I_{p_\parallel}/I'_{p_\parallel})_{max}$ seem to be predicted rather satisfactorily.

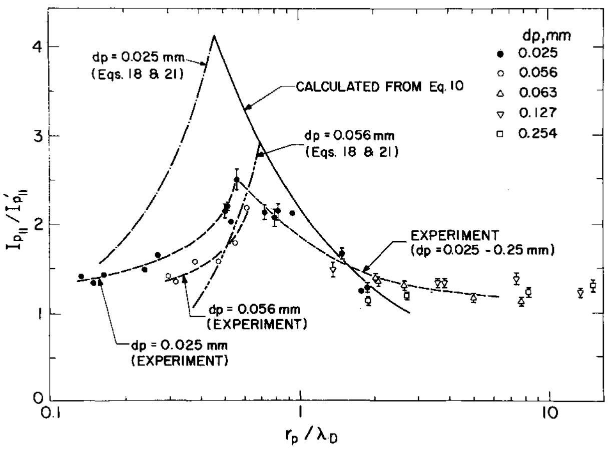

Fig. 9 Normalized amplitude of ion current peak vs r_p/λ_D. $\eta_p = -15$.

The results just presented appear to support the proposed physical model and the explanation of the ion current peak in terms of ion-neutral encounters in the region of small electric fields beyond the sheath. The ion-ion collisions, possibly do not contribute, per se, in any significant measure to the ion current peak. However, a very small λ_{ii}, equivalent to a continuously fluctuating ion path, may play some role in the entire process by effectively increasing the ion-neutral collision cross section.

A natural consequence of the suggested physical model is a conclusion that the shape and the width of the ion current peak are expected to depend on the ion temperature, T_i, and ion Mach number, S. A decrease in T_i and an increase in S should produce a narrowing of the peak because under such conditions even a small deviation from $\theta = 0°$ may be enough to allow most of the ions entering the field to escape its influence. Thus a large width of the ion current peak observed by Sonin can be explained in terms of higher T_i and lower S in his experiments as compared to those in this work. Another consequence of ion-neutral encounters is the absence of an end effect even with relatively short probes and small r_p/λ_D. At the conditions of: a) very large mean free paths, b) $r_p/\lambda_D < 1$, c) high-speed flow, and d) not very large l_p/d_p, an aligned cylindrical probe is bound to experience a very strong end effect because most of the ions collected by the probe enter the field through the front area of the sheath.‡ Collisions destroy, of course, this undisturbed, "ordered" flow pattern of ions and cause most of the ions to be collected through the sides of the sheath as in a no end-effect case.

If the explanation of the ion current peak, presented here, is accepted, then commonly used criteria for a collisionless flow ($\lambda > r_p$) should be modified for the case of a long wire probe aligned with the flow velocity. A condition that λ_{in} be larger than an average distance (free of collisions) traveled by an ion within the electric field region can probably be satisfied in most cases by demanding that λ_{in} be much larger than the sheath thickness, $\lambda_{in} \gg r_s$.

Probe Response in the OML Region

The theory of Smetana and Eq. (3) seem to describe quite well the probe response in the range $r_p/\lambda_D < 0.5$. When $u \sin\theta \gg (2kT_i/m_i)^{1/2}$, the dependence $I_p = f(\theta)$ can be approximately described by Eq. (2). The theory of Laframboise seems to be valid in the entire OML region explored in this study. This last conclusion differs from the results and conclusions reported in the literature. On the other hand, the reported results differ between themselves in some respects.[1,11,14,16] It is suggested that at least part of the apparent disagreement can be explained in terms of ion-neutral collisional effects described in this paper.

Consider first the results of an important study reported by Sonin.[1] Using readings obtained with aligned probes he found that at $r_p/\lambda_D < 2.8$ the current increased significantly above the theoretical predictions. From the test conditions in his experiments we conclude that the condition $\lambda_{in} \gg r_s$ was not satisfied, and, therefore, aligned probes were likely subject to collisional effects described in this paper. Data presented in Table 5.1 of Ref. 1 show that the ratio $I_{p_\parallel}/I_{p_\perp}$ (in the notation used in this paper) increases significantly at low values of $(r_p/\lambda_D)^2\ I_i$ (based on measurements with aligned probes) and consequently at low values of r_p/λ_D. This may indicate an increase in the ion current peak as r_p/λ_D decreases toward its lower limit explored in Sonin's work ($r_p/\lambda_D \approx 1$). If this increase is due to collisional effects then the value of a "collisionless" current constitutes only a fraction of the measured current $I_{p_\parallel}$ and this fraction decreases as r_p/λ_D is reduced. As a result, if inferred values of collisionless current were used for interpretation of the probe response, the values of the dimensionless current I_i would decrease significantly at lower values of $(r_p/\lambda_D)^2\ I_i$ and would tend to level off somewhere around $(r_p/\lambda_D)^2 I_i \approx 40$, i.e., in the orbital-motion limit. (Mapping the values $I_{p_\perp}/I_{p_\parallel}$ vs $(r_p/\lambda_D)^2 I_i$ shows a change in the slope in the neighborhood of $(r_p/\lambda_D)^2\ I_i \approx 40$.) Also, if only a fraction of $I_{p_\parallel}$ were used for evaluation of the probe response, the data points indicated in Fig. 5.2 (Ref. 1) would be additionally shifted toward lower values of $(r_p/\lambda_D)^2 I_i$. This correction is relatively small at large $(r_p/\lambda_D)^2 I_i$, but it may become pronounced at low $(r_p/\lambda_D)^2 I_i$. We note that both the reduction of I_i and shifting the experimental points toward lower $(r_p/\lambda_D)^2 I_i$ may possibly bring the data not only into an agreement with theory in the OML region but, perhaps, also into an even closer agreement in the "space-charge-limited" region. The latter remark refers to the measurements obtained with probe 16 in the runs 34-1, 35-2, 35-3, and 35-6 (Ref. 1).

Consider next the results published by Dunn and Lordi.[15,16] Their data obtained with aligned probes and assumed to be unaffected by flow velocity extended over a range $0.5 < r_p/\lambda_D < 2$. For $r_p/\lambda_D > 1$, the free-molecular probe data compared favorably with theory of Laframboise, but at smaller r_p/λ_D, the probe current was higher than theoretical prediction. The electron densities obtained with probes of different diameters were different and thinner probes indicated consistently higher densities. We may attempt to explain these results in terms of ion-neutral collisions. From the test conditions in Dunn and Lordi's experiments, it may be inferred that λ_{in} was of the same order or somewhat larger than sheath radius, r_s, and, according to the model suggested in this paper, aligned probes were subject to collisional effects. This inference is consistent with insensitiveness to probe length, found by Dunn and Lordi. Now, the ion current peak of an aligned probe increases rapidly when r_p/λ_D decreases from about 2 to 0.5 (compare Fig. 9) and this may explain both a departure of experimental values from theory (at $r_p/\lambda_D < 1$) and disagreement between electron densities measured with probes of different diameters. Moreover, a comparative examination of Fig. 5 in Ref. 16 and Fig. 9 in this paper shows an essential resemblance in the variation of the ion current at $r_p/\lambda_D < 1$. Both graphs display a sharp peak in the narrow range of $0.5 < r_p/\lambda_D < 1$. The agreement is quite satisfactory considering that the results were obtained at quite different experimental conditions.

Finally, consider the shock-tunnel study of Lederman, Bloom, and Widhopf.[14] They found that the ion current in the OML region was much higher than predicted by Laframboise's theory, and that the current density increased with the decrease of l_p/d_p ratio. The ion-neutral mean free path was relatively large ($\lambda_{in} \approx 6$ mm) and, at least in most of the tests, it was much larger than r_s. This fact combined with small r_p/λ_D (0.03–5) and high-flow velocity ($S = 3.4$) in Lederman's experiments explains the sensitivity to probe length which can be identified as an end effect. This effect could be, of course, responsible for very large deviations of experimental results from theory of Laframboise.

6. Conclusions

The main conclusions derived from the study are as follows.

1) The angle of incidence of a cylindrical probe immersed in a plasma stream moving with a hyperthermal velocity (hyperthermal w.r.t. ions) may have a significant effect on the response of the probe. This effect is strongly influenced by the probe radius-to-Debye length ratio (r_p/λ_D) and it may depend on the ion temperature, ion Mach number and probe potential. At small and intermediate values of r_p/λ_D, a probe which at large angles of attack operates in an essentially collisionless regime ($\lambda_{\text{charge-neutral}} > d_p$), after being brought into an alignment with the flow, may enter a transition regime even when the relevant mean free paths are many times larger than r_p. On the other hand, a probe which operates in a highly rarefied plasma (ionospheric environment, $\lambda_{\text{charge-neutral}} \gg r_p$) when turned into an aligned position may be subject to a strong end effect (particularly when $r_p/\lambda_D < 1$).

2) In almost all experiments of this investigation, an ion current increase has been observed near zero angle of incidence. The magnitude of this effect has been found to depend very strongly on r_p/λ_D and has been very significant for $0.5 < r_p/\lambda_D < 2$.

3) The effect of an ion current peak and, for that matter, the entire $I_p = f(\theta)$ characteristic has been found insensitive to the probe length provided the length-to-diameter ratio is larger than a certain minimum (probably around 40 in these experiments). The absence of an end effect appears to be closely related to the presence of ion collisions.

4) A physical model has been suggested which explains the effect of the ion current peak in terms of ion-neutral collisions in the electric field region surrounding the probe. Simple theoretical treatment is presented that predicts the ion current peak at small r_p/λ_D and its dependence on the probe radius. A complete quantitative verification will require a substantial progress in predicting the sheath size and frequency of ion-neutral collisions in the field around the probe.

‡ For a quantitative description of end and convection effects, the reader is referred to Refs. 18, 19, and 28.

5) The structure of the current peak is expected to depend on the ion temperature, T_i, and hyperthermal velocity of ions u. Under certain conditions and experimental procedures, the effect of the current peak may be, probably, utilized for estimating sheath radius, ion temperature, and transport properties of plasma.

6) If collisional effects are to be avoided for an aligned probe at high-speed flow and large λ_D, then charge-neutral mean free path should be much larger than the sheath thickness ($\lambda \gg r_s$).

7) Smetana's calculations and Eq. (3) describe quite well the response of a probe, if its diameter is smaller than the Debye length ($d_p < \lambda_D$). Laframboise's theoretical predictions seem to be accurate at any r_p/λ_D *including the OML region.* Disagreements with his theory reported in previous experiments can be explained either in terms of ion-neutral collisions associated with aligned probe orientation (in "near collision-free" plasmas) or in terms of an end effect (in highly rarefied plasmas).

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